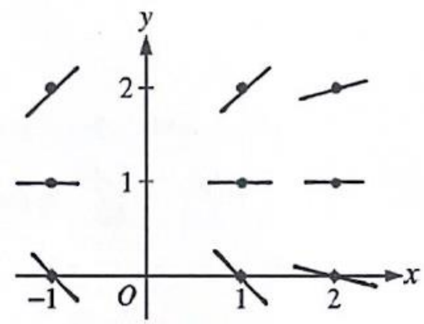
2008 #5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



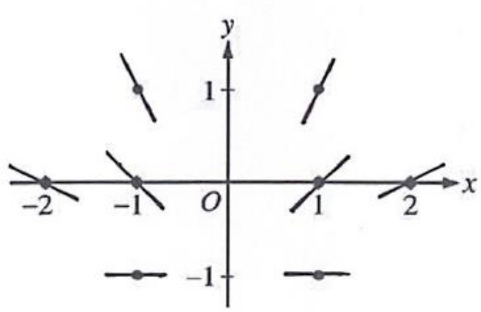
(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

2006 #5

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

2008 #5

b) $\frac{dy}{dx} = \frac{y-1}{x^2}$

$\frac{1}{y-1}\,dy = x^{-2}\,dx$

$\ln|y-1| = -x^{-1} + C$

$|y-1| = Ce^{-1/x}$

$y = 1 + Ce^{-1/x}$

$0 = 1 + Ce^{-1/2}$

$-1 = \frac{C}{\sqrt{e}}$

$-\sqrt{e} = C$

$y = 1 - \sqrt{e}\cdot e^{-1/x}$

$y = 1 - e^{\frac{1}{2}-\frac{1}{x}}$

c) $\lim_{x\to\infty}\left[1 - e^{1/2 - \frac{1}{x}}\right] = 1 - e^{1/2}$

2006 #5

b) $\frac{dy}{dx} = \frac{1+y}{x}$

$\frac{1}{y+1}\,dy = \frac{1}{x}\,dx$

$\ln|y+1| = \ln|x| + C$

$|y+1| = e^{\ln|x| + C}$

$y+1 = Ce^{\ln|x|}$

$y = -1 + Ce^{\ln|x|}$

$y = -1 + C|x|$

$1 = -1 + C$

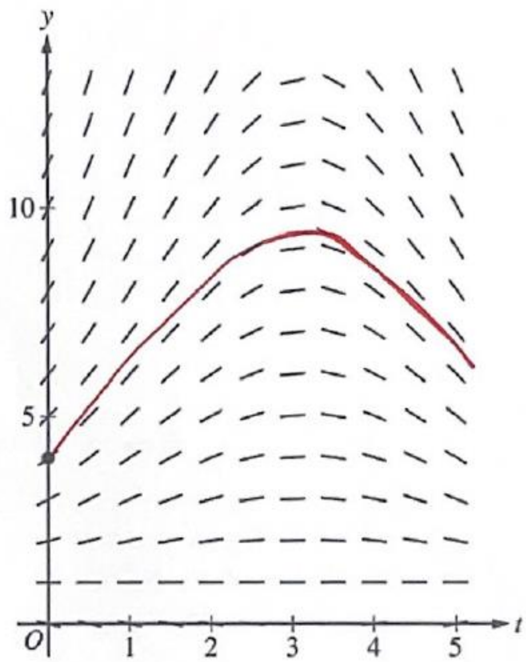
$2 = C$

$f(x) = -1 + 2|x|, \quad x < 0$

2024 #3

3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

2024 #3

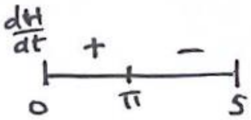
b) $H(t) > 1$

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right) = 0$$

$$\cos\left(\frac{t}{2}\right) = 0$$

$$\frac{t}{2} = \frac{\pi}{2}$$

$$t = \pi$$



H has a max when $t = \pi$,
b/c $\frac{dH}{dt}$ changes signs from + to −.

c) $$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

$$\frac{1}{H-1}\,dH = \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$$

$$\ln|H-1| = \sin\left(\frac{t}{2}\right) + C$$

$$|H-1| = Ce^{\sin\left(\frac{t}{2}\right)}$$

$$H = 1 + Ce^{\sin\left(\frac{t}{2}\right)}$$

$$4 = 1 + Ce^{0}$$

$$3 = C$$

$$H(t) = 1 + 3e^{\sin\left(\frac{t}{2}\right)}$$